



**MATHEMATICS  
HIGHER LEVEL  
PAPER 1**

Friday 9 November 2001 (afternoon)

2 hours

Name

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Number

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**INSTRUCTIONS TO CANDIDATES**

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Calculator

| Make | Model |
|------|-------|
|      |       |

| EXAMINER  | TEAM LEADER | IBCA      |
|-----------|-------------|-----------|
| TOTAL /60 | TOTAL /60   | TOTAL /60 |

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.

1. A coin is biased so that when it is tossed the probability of obtaining heads is  $\frac{2}{3}$ . The coin is tossed 1800 times. Let  $X$  be the number of heads obtained. Find
- (a) the mean of  $X$ ;
  - (b) the standard deviation of  $X$ .

|                 |   |
|-----------------|---|
| <i>Working:</i> | <i>Answers:</i><br>(a) _____<br>(b) _____ |
|-----------------|---|

2. The complex number  $z$  satisfies  $i(z + 2) = 1 - 2z$ , where  $i = \sqrt{-1}$ . Write  $z$  in the form  $z = a + bi$ , where  $a$  and  $b$  are real numbers.

|                 |                         |
|-----------------|-------------------------|
| <i>Working:</i> | <i>Answer:</i><br>_____ |
|-----------------|-------------------------|

3. The polynomial  $f(x) = x^3 + 3x^2 + ax + b$  leaves the same remainder when divided by  $(x - 2)$  as when divided by  $(x + 1)$ . Find the value of  $a$ .

*Working:*

*Answer:*

4. Consider the infinite geometric series

$$1 + \left(\frac{2x}{3}\right) + \left(\frac{2x}{3}\right)^2 + \left(\frac{2x}{3}\right)^3 + \dots$$

- (a) For what values of  $x$  does the series converge?
- (b) Find the sum of the series if  $x = 1.2$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

5. The function  $f: x \mapsto \frac{2x+1}{x-1}$ ,  $x \in \mathbb{R}$ ,  $x \neq 1$ . Find the inverse function,  $f^{-1}$ , clearly stating its domain.

*Working:*

*Answer:*

6. If  $A = \begin{pmatrix} x & 4 \\ 4 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & y \\ 8 & 4 \end{pmatrix}$ , find the values of  $x$  and  $y$ , given that  $AB = BA$ .

*Working:*

*Answers:*

7. The line  $y = 16x - 9$  is a tangent to the curve  $y = 2x^3 + ax^2 + bx - 9$  at the point  $(1, 7)$ .  
Find the values of  $a$  and  $b$ .

*Working:*

*Answers:*

8. A continuous random variable  $X$  has probability density function

$$f(x) = \begin{cases} \frac{4}{\pi(1+x^2)}, & \text{for } 0 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find  $E(X)$ .

*Working:*

*Answer:*

9. The matrix  $\begin{pmatrix} 1 & -2 & -3 \\ 1 & -k & -13 \\ -3 & 5 & k \end{pmatrix}$  is singular. Find the values of  $k$ .

*Working:*

*Answers:*

10. Consider the function  $y = \tan x - 8 \sin x$ .

(a) Find  $\frac{dy}{dx}$ .

(b) Find the value of  $\cos x$  for which  $\frac{dy}{dx} = 0$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

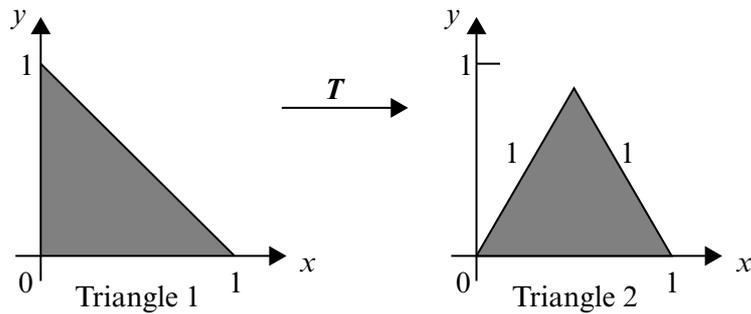
(b) \_\_\_\_\_

11. Find the values of  $x$  for which  $|5 - 3x| \leq |x + 1|$ .

*Working:*

*Answers:*

12. A linear transformation  $T$  maps Triangle 1 to Triangle 2, as shown in the diagram.



Find a matrix which represents  $T$ .

*Working:*

*Answer:*

13. Consider the tangent to the curve  $y = x^3 + 4x^2 + x - 6$ .

(a) Find the equation of this tangent at the point where  $x = -1$ .

(b) Find the coordinates of the point where this tangent meets the curve again.

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

14. A point  $P(x, x^2)$  lies on the curve  $y = x^2$ . Calculate the minimum distance from the point  $A\left(2, -\frac{1}{2}\right)$  to the point P.

*Working:*

*Answer:*

15. Point  $A(3, 0, -2)$  lies on the line  $\mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ , where  $\lambda$  is a real parameter. Find the coordinates of **one** point which is 6 units from  $A$ , and on the line.

*Working:*

*Answer:*

16. Let  $\theta$  be the angle between the unit vectors  $\mathbf{a}$  and  $\mathbf{b}$ , where  $0 < \theta < \pi$ . Express  $|\mathbf{a} - \mathbf{b}|$  in terms of  $\sin \frac{1}{2}\theta$ .

*Working:*

*Answer:*

17. How many four-digit numbers are there which contain at least one digit 3?

*Working:*

*Answer:*

18. The probability that a man leaves his umbrella in any shop he visits is  $\frac{1}{3}$ . After visiting two shops in succession, he finds he has left his umbrella in one of them. What is the probability that he left his umbrella in the second shop?

*Working:*

*Answer:*

19. A sample of radioactive material decays at a rate which is proportional to the amount of material present in the sample. Find the half-life of the material if 50 grams decay to 48 grams in 10 years.

*Working:*

*Answer:*

20. Find the area enclosed by the curves  $y = \frac{2}{1+x^2}$  and  $y = e^{\frac{x}{3}}$ , given that  $-3 \leq x \leq 3$ .

*Working:*

*Answer:*



